Optimal weighting scheme for suppressing cascades and traffic congestion in complex networks

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This paper is motivated by the following two related problems in complex networks: (i) control of cascading failures and (ii) mitigation of traffic congestion. Both problems are of significant recent interest as they address, respectively, the security of and efficient information transmission on complex networks. Taking into account typical features of load distribution and weights in real-world networks, we have discovered an optimal solution to both problems. In particular, we shall provide numerical evidence and theoretical analysis that, by choosing a proper weighting parameter, a maximum level of robustness against cascades and traffic congestion can be achieved, which practically rids the network of occurrences of the catastrophic dynamics.

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I. INTRODUCTION

Large networked systems are the basic support of modern infrastructures and protecting them from random failures or intentional attacks is an active topic of research in network science [1–6]. Instances of breakdown with severe consequences include large-scale blackouts of power grids [7] and heavy congestions on the Internet [8]. Due to the complex topology of the network, breakdown on a global scale can be triggered by small, local failures through the mechanism of cascading [2,4,5,9–12]. Articulating control strategies to prevent complex networks from cascading breakdown becomes then a pertinent issue.

In Ref. [3], a method is proposed whereby a set of "insignificant" nodes that contribute more load to the network than they handle is removed to enhance the overall load-handling capability of the network. This strategy may be regarded as "hard" because it requires that certain nodes be removed from the network, which leads to structural changes in the network. An issue of interest is whether some proper "soft" control strategy can be developed to prevent cascading breakdown but to keep the connections among nodes unchanged.

In this paper, we present a general "soft" control strategy to address cascading breakdown and traffic congestion in a unified manner. Our main idea is based on the following two considerations: (i) in real-world networks the node capacity is not linearly proportional to the load, and (ii) transmission paths can be adjusted by arbitrarily given link weights. For the capacity-load relation, a recent work [13] indicates that in real-world networks, there are deviations from the simple proportional relation between the initial load and capacity, although it has been used widely in existing works on cascading dynamics [2,4,5,9–12]. For a complex network, different nodes can have different degrees. Links to and from hub nodes tend to be used more frequently than other links in the network. The weight of a link can thus be assumed to depend on the degrees of the two nodes that it connects such that loads bypassing links and nodes can be tuned. Consequently, information flows on the network depend on the weights. The finding of this paper is that there exists an optimal weighting scheme for which cascading failures and traffic congestion can be suppressed significantly. In particular, the robustness of a network against cascading failures is characterized by the critical values of a pair of tolerance parameters, at which there exists a phase transition from an absorbing (free) state to a cascading state. The critical values can be regarded, qualitatively, as corresponding to the minimum cost for protecting networks to avoid cascading damages. The optimal weighting scheme can thus be quantified by the lowest minimum cost. For traffic flow dynamics, the network throughput is characterized by the maximum packet generation rate for which the network is free of congestion. The higher the maximum generation rate, the more efficient for network traffic. What we have found through heuristic analysis and numerical computations on both model and real-world networks is that under the optimal weighting scheme, the lowest minimum protection cost and the highest packet generation rate can be achieved simultaneously and, quite strikingly, the minimum cost can be several orders of magnitude smaller than the values realized in the underlying nonweighted network. In a practical sense, this means that the network can essentially be cascade- and congestion-free through the control implementation of some appropriate weighting scheme.

In Sec. II, we present our model for cascading dynamics on weighted complex networks, taking into account features of load dynamics that are typical of real-world networks. In Sec. III, we provide numerical evidence for the existence of an optimal weighting scheme that results in the maximum amount of robustness against cascading. A heuristic analysis is provided in Sec. IV for estimating the optimal weighting parameter and quantities characterizing a network's resilience to cascading failures and traffic congestion. Conclusions and a discussion are presented in Sec. V.

II. CASCADING MODEL BASED ON WEIGHTING SCHEME AND REALISTIC LOAD-CAPACITY RELATION

To take into account the relative importance of various links for transmission in the network, we assume the following weight for the link between an arbitrary pair of nodes:

$$w_{ij}=A_{ij}(k_ik_j)^{\theta}, \quad (1)$$

where k_i and k_j are the degrees of nodes i and j, respectively, θ is an adjustable control parameter, and A is the adjacency matrix of the network ($A_{ij}=1$ if nodes i and j are connected, $A_{ij}=0$ otherwise, and $A_{ii}=0$). A weighted path from node a to b can be specified completely by all nodes located on the path from a to b in order: $a\equiv 1,2,\ldots,n\equiv b$. The weighted path length is the sum of link weights from a to b: $d_{a\to b}\equiv\Sigma_{i=1}^{n-1}w_{i,i+1}$, from which the shortest weighted path length can be obtained. The load at a node can be defined as the total number of *weighted* shortest paths passing through the node. The capacity of a node is the maximum load that the node can handle. While previous studies on cascades in networks [2,14] assume that the capacity of a node is directly proportional to its initial load, a recent empirical study [13] indicates that real-world networked systems tend to have a larger unoccupied portion of the capacities on nodes with smaller capacities. Following Ref. [13], we assume the following relationship between the node capacity C_i and the initial load L_i:

$$C_i=\alpha+\beta L_i, \quad i=1,2,\ldots N, \quad (2)$$

where $\alpha\geqslant 0$ and $\beta\geqslant 0$ are the capacity parameters and N is the initial number of nodes. Equation (2) models quite well the empirical capacity-load relation for a number of real-world networks [13].

We consider the type of attacks that remove a single node with the highest load, since the failures of such a node can affect significantly loads at other nodes. The loads at some nodes may exceed their capacities and failures occur consequently. The loads are globally redistributed in the network. When more nodes fail, the weighted shortest paths among all pairs of nodes and the loads are then recalculated according to the initially assigned edge weights associated with the structural changes. The process of node failure and load redistribution is repeated until no node fails, at which point the cascading process can be regarded as being completed. For sufficiently large values of α and β, load redistribution triggered by attacks is unlikely to cause a cascading breakdown. We expect that, as α or β is reduced, there exist critical points α_c (for fixed β) and β_c (for fixed α), below which cascading failures occur. For fixed β (or α), there should then exist a transition at α_c (or β_c) from an "absorbing" to an "active" phase as α (or β) is decreased. The critical points α_c and β_c can be regarded as "robustness parameters" for the network. The smaller the values of α_c and β_c, the more robust is the network because cascading failures will not occur even when α and β are small (insofar as the conditions $\alpha\geqslant\alpha_c$ and $\beta\geqslant\beta_c$ are satisfied). It thus suffices to focus on how the weighting parameter θ affects the network robustness as characterized by α_c and β_c.

III. NUMERICAL EVIDENCE FOR OPTIMAL WEIGHTING STRATEGY

A convenient criterion to determine the onset of cascading failures is to examine the load redistribution after the node of the largest load is disabled. After the attack, the load at node

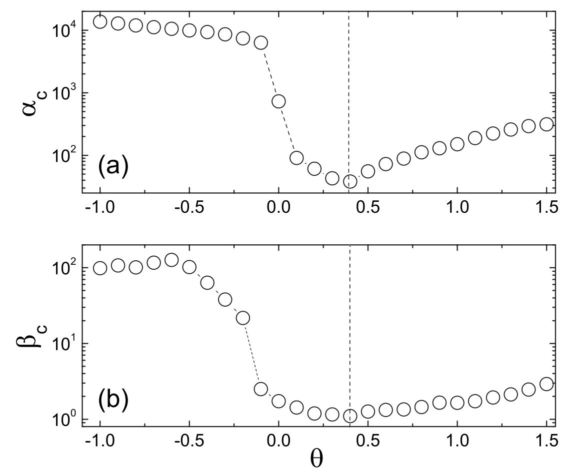

FIG. 1. For model scale-free networks, robustness parameters (a) α_c (for fixed $\beta=1.0$) and (b) β_c (for fixed $\alpha=1.0$) vs the weighting parameter θ. The vertical dashed lines indicate the existence of an optimal value $\theta=\bar{\theta}\approx 0.4$ at which both α_c and β_c are minimized. The results are obtained by using 100 runs of network dynamics according to load redistribution for each of the 100 network realizations. Network size is 1000 and the minimum of degree is $k_{min}=10$.

j changes from L_j to L_j'. A cascading process occurs if $L_j'>C_j=\alpha+\beta L_j$ for $j=1,\ldots,N$, which requires, for a fixed value of β, $L_j'-\beta L_j>\alpha$. If $L_j'-\beta L_j<\alpha$ for all j, there will be no cascading. The critical values α_c and β_c for the onset of cascading can then be determined by

$$\alpha_c=\max(L_j'-\beta L_j|j=1,2,\ldots,N),$$

$$\beta_c=\max[(L_j'-\alpha)/L_j|j=1,2,\ldots,N]. \quad (3)$$

Simulation results of α_c and β_c for different values of the weighting parameter θ on scale-free networks generated by the Barabási-Albert algorithm [17] are shown in Fig. 1. We observe that, as θ is increased from some negative value, both α_c and β_c decrease. There exists an optimal value $\theta=\bar{\theta}\approx 0.4$ about which both α_c and β_c are minimized, indicating that the network is maximally resistant to cascading failures. Comparing with the case of nonweighted links ($\theta=0$), α_c can be reduced by over one order of magnitude [Fig. 1(a)] and the reduction in β is also quite significant [Fig. 1(b)].

The existence of an optimal weighting strategy to maximize the network resistance to cascading dynamics has also been observed in real-world networks. Here we present two examples: the Internet and power grids. Figure 2 shows the variations of α_c and β_c with θ for the Internet at the level of autonomous systems, which contains a strong scale-free component [15]. Compared with the $\theta=0$ case (unweighted routing strategy), for $\theta\approx\bar{\theta}$, the value of α_c can be reduced by nearly three orders of magnitude [Fig. 2(a)]. Optimization can also occur with respect to β, as shown in Fig. 2(b). Similar results have been obtained for the power grid of the western United States, as shown in Figs. 3(a) and 3(b). Utilizing optimal weighting to significantly enhance a network's

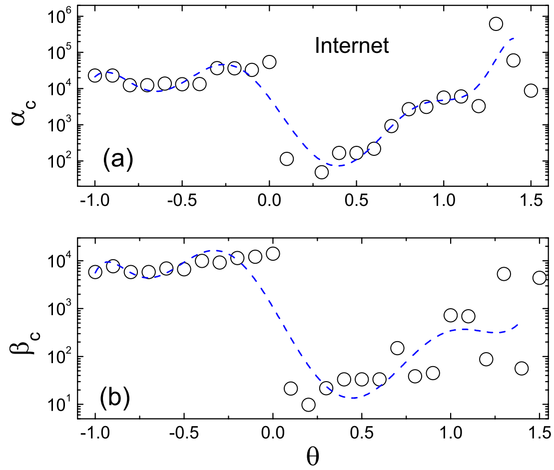

FIG. 2. (Color online) For the Internet at the level of autonomous systems [15], (a) α_c (for fixed β=20.0) and (b) β_c (for fixed α=20.0) vs the weighting parameter θ. The original network has 22 963 nodes and average degree is approximately 4.22. Due to the relatively large size of the network, in our simulations all nodes of degree one have been removed. The resulting network has 15 123 nodes and the average degree is about 5.37. The dashed line is the polynomial fitting. The existence of an optimal weighting parameter at which both α_c and β_c are minimized can be seen.

ability to resist catastrophic dynamics thus appears to be generally viable.

In order to understand the behaviors in Figs. 1–3, we examine the weighted betweenness $B(i)$ of an arbitrary node i, which as a function of the node degrees k is defined as

$$B(k) = \sum_{i,k_i=k} B(i)/N_k,$$

where N_k is the number of nodes of degree k. The onset of cascading failures is usually triggered by the failure of the node with the maximum load. The higher the load that a failed node carries, the more difficult it is for the extra load to be absorbed by the remaining nodes in the network. There should then be a positive correlation between the maximum load and values of α_c and β_c. Figure 4 shows the results of the maximum node load $\bar{B}_m$ as a function of θ, for the same scale-free network as in Fig. 1. We see that $\bar{B}_m$ vs θ displays similar behaviors to those of α_c and β_c. In particular, there exists an optimal value $\bar{\theta}$ for which $\bar{B}_m$ reaches minimum. The estimated values of $\bar{\theta}$ from Figs. 1 and 4 are essentially the same, indicating a strong correlation between $\bar{B}_m$ and (α_c, β_c). It is thus reasonable to focus on the dependence of $\bar{B}_m$ on θ.

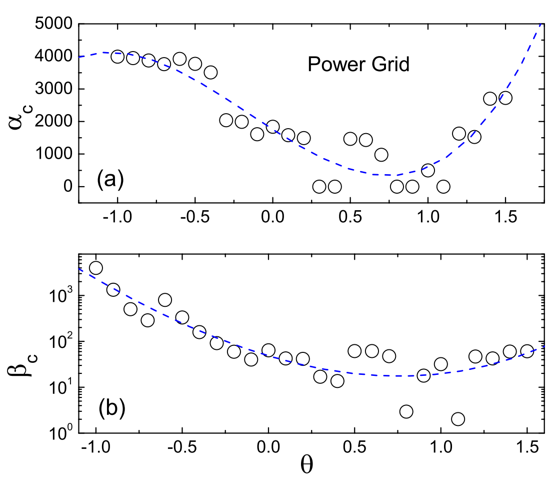

FIG. 3. (Color online) For the Northwestern US power transmission grid [16], (a) α_c (for fixed β=20.0) and (b) β_c (for fixed α=20.0) vs the weighting parameter θ. The network has 4941 nodes and the average degree is about 2.67. The dashed line is the polynomial fitting.

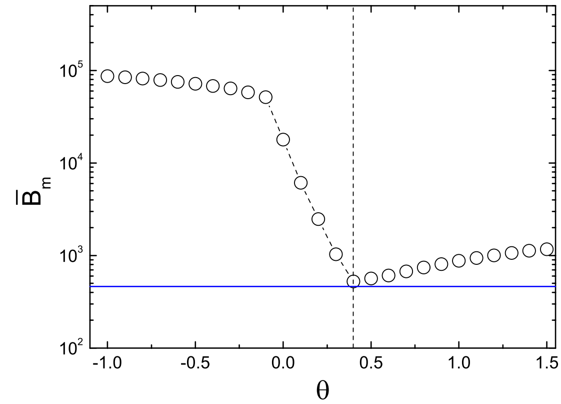

FIG. 4. (Color online) For the same scale-free network as in Fig. 1, $\bar{B}_m$ vs the weighting parameter θ. Apparently, $\bar{B}_m$ reaches minimum at the same optimal value $\bar{\theta}$ for which (α_c, β_c) are minimized, indicating a strong correlation between the former and the latter. The horizontal line indicates our analytically estimated value of $\bar{B}_m(\bar{\theta})$.

IV. THEORY

The purposes of our analysis are (i) to estimate the value of $\bar{B}_m(\bar{\theta})$, (ii) to determine the value of the optimal weighting parameter $\bar{\theta}$, and (iii) to establish a connection between cascading dynamics and traffic congestion.

A. Estimation of $\bar{B}_m(\bar{\theta})$

For an unweighted network, it is known that the distribution of the betweenness $B(k)$ is highly heterogeneous and obeys the following algebraic scaling law:

$$B(k) \sim k^{\alpha}, \tag{4}$$

where $\alpha > 0$ is the scaling exponent. An example illustrating the scaling is shown in Fig. 5(a), where the scaling exponent is $\alpha \approx 1.9$, in consistency with previous results [18]. Figure 5(b) shows, for a weighted network, the dependence of the betweenness B_k on the node degree for $\theta = \bar{\theta}$. We see that B_k is approximately constant for most nodes in the network, except for a small set of nodes with relatively large degrees. Figure 5(c) shows a typical profile of the total betweenness $B_{sum}(\theta)$, which is approximately constant for all values of θ

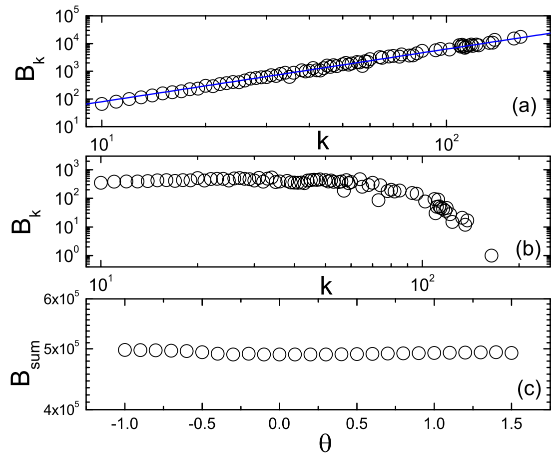

FIG. 5. (Color online) For a standard scale-free network of $N=1000$ nodes, (a) betweenness distribution for $\theta=0$, (b) the distribution for the optimal weighting scheme $\theta=\bar{\theta}$, and (c) total betweenness of all nodes in the network versus θ. Other network parameters are the same as in Fig. 1.

of interest. We observe that $B_{sum}(\bar{\theta})\approx B_{sum}(\theta=0)$, which allows us to write

$$B_{sum}(\theta=0) = N\int_{k_{min}}^{k_{max}} B(k)P(k)dk,$$

where $B(k)$ is evaluated for $\theta=0$, and k_{min} and k_{max} are the minimum and the maximum degree, respectively. For a BA scale-free network, the degree distribution is $P(k)=2k_{min}^2k^{-3}$ and the maximum degree is given by $k_{max}=k_{min}\sqrt{N}$. Since for the optimal value $\bar{\theta}$ the load is approximately constant with respect to node degrees except a few very large degree nodes [cf. Fig. 5(b)], we have

$$\bar{B}_m(\bar{\theta}) \approx \frac{1}{N}B_{sum}(\theta=0) = 20k_{min}^{1.9}(1-N^{-0.05}). \quad (5)$$

As shown in Fig. 4, the horizontal line estimated using the prediction of Eq. (5) agrees well with the numerically obtained value of $\bar{B}_m$ achieved for $\theta=\bar{\theta}$.

B. Optimal weighting parameter $\bar{\theta}$

We now turn to estimating the optimal weighting parameter $\bar{\theta}$. For node i with degree k_i, the number of weighted shortest paths passing through it can be calculated by the sum of shortest paths passing through all neighbors of node i, i.e.,

$$2B_i = \sum_{j=1}^{N} A_{ij}B_{j\to i}, \quad (6)$$

where $B_{j\to i}$ denotes the number of shortest paths from j to i. The quantity $B_{j\to i}$ can be estimated as

$$B_{j\to i} = B_j\frac{w_{ij}^{-1}}{\sum_{l=1}^{N} w_{jl}^{-1}}, \quad (7)$$

where B_j is the number of shortest paths through node j. Equation (7) states the fact that the number of shortest paths from a node to a neighboring node is approximately inversely proportional to the weight of the link connecting the two nodes. If the weight is large, fewer shortest paths are likely to pass between them. We thus have

$$2B_i = \sum_{j=1}^{N} A_{ij}B_j\frac{[(k_ik_j)^{\theta}]^{-1}}{\sum_{l=1}^{N} A_{jl}[(k_jk_l)^{\theta}]^{-1}}, \quad (8)$$

where the summation in the denominator can be written as

$$k_j^{-\theta}\sum_{l=1}^{N} A_{jl}k_j^{-\theta} = k_j^{1-\theta}\sum_{k'=k_{min}}^{k_{max}} P(k'|k_j)(k')^{-\theta},$$

where $P(k'|k_i)$ is the conditional probability that a node of degree k_i has a neighbor of degree k'. For a network without degree-degree correlation, we have $P(k'|k_i)=k'P(k')/\langle k\rangle$. We thus obtain

$$\sum_{l=1}^{N} A_{jl}[(k_jk_l)^{\theta}]^{-1} = k_j^{1-\theta}\sum_{k'=k_{min}}^{k_{max}} P(k'|k_i)k'^{-\theta}$$
$$= k_j^{1-\theta}\sum_{k'=k_{min}}^{k_{max}}\frac{k'^{1-\theta}P(k')}{\langle k\rangle} = \frac{k_j^{1-\theta}\langle k^{1-\theta}\rangle}{\langle k\rangle}, \quad (9)$$

where the identity

$$\sum_{k'=k_{min}}^{k_{max}} k'^{1-\theta}P(k') = \langle k^{1-\theta}\rangle$$

has been used. Moreover, as shown in Fig. 5(b), for most nodes except a few very large degree nodes, their loads are approximately equal: $B_1\approx B_2\approx\cdots\approx B_k\approx\bar{B}_m$. This leads to

$$2\bar{B}_m \approx \sum_{j=1}^{N} A_{ij}\bar{B}_m\frac{(k_ik_j)^{-\bar{\theta}}\langle k\rangle}{k_j^{1-\bar{\theta}}\langle k^{1-\bar{\theta}}\rangle} = \frac{\langle k\rangle\bar{B}_m}{\langle k^{1-\bar{\theta}}\rangle}k_i^{1-\bar{\theta}}\sum_{k'=k_{min}}^{k_{max}} P(k'|k_i)k'^{-1}. \quad (10)$$

We thus have

$$k_i^{1-\bar{\theta}}/\langle k^{1-\bar{\theta}}\rangle = 2,$$

where

$$\langle k^{1-\bar{\theta}}\rangle = \int_{k_{min}}^{k_{max}} 2k_{min}^2k^{-3}k^{1-\bar{\theta}}dk = \frac{2}{1+\bar{\theta}}k_{min}^{1-\bar{\theta}}.$$

The optimal weighting parameter $\bar{\theta}$ can be determined implicitly by

$$\left(\frac{k_i}{k_{min}}\right)^{1-\bar{\theta}} = \frac{4}{1+\bar{\theta}}. \qquad (11)$$

Since this result is based on a mean-field type of approximation, k_i should be large to warrant accurate estimate of $\bar{\theta}$. However, as indicated by Fig. 5(a), k_i should be in the flat range of B_k, so that the condition $B_i \approx \bar{B}_m$ can be satisfied. For the numerical example in Figs. 4 and 5 a reasonable choice is $k_i=60$. Since $k_{min}=10$, we obtain $\bar{\theta} \approx 0.42$, which agrees well with the value of 0.4 obtained from direct numerical simulation.

C. Relation between cascading dynamics and traffic congestion

There is an underlying relation between cascading dynamics and congestion of information traffic. In particular, assume that packets are generated with probability R at each node and are routed along the shortest paths from their origins to destinations. According to queueing theory [19], in a free flow state, the average queue length $\langle l_i \rangle$ of node i is given by $\langle l_i \rangle = c_i/(1-c_i)$, where $c_i < 1$ is the average number of packets passing through i in unit time. Previous works have demonstrated that c_i is proportional to the betweenness B_i [20,21]: $c_i = RB_i/(N-1)$. When buffer size is finite, if any l_i exceeds the maximum queue length l_{max}, congestion will occur. The critical generation rate R_c that defines the onset of traffic congestion can then be estimated as [22]

$$R_c = l_{max}(N-1)/[(l_{max}+1)B_{max}] \approx (N-1)/B_{max}.$$

The critical rate R_c in fact measures the throughput of a network in handling information traffic. The higher R_c is, the more resilient a network is to traffic congestion. A network with the lowest maximum node betweenness $\bar{B}_m$ thus has the maximum throughput. Setting $\theta = \bar{\theta}$ results in the lowest possible value for $\bar{B}_m$. This not only yields the maximum degree of network robustness against cascading failures, but also enhances the transmission efficiency in routing traffic along the weighted shortest paths to avoid congestion, e.g., in a transportation network. It should be noted that our analysis based on the queueing theory is only applicable for the topological shortest-path routing algorithm, so the fact that attaining the maximum throughput at $\theta = \bar{\theta}$ of the weighting scheme is restricted to the weighted shortest-path routing based on the global topological information other than the routing algorithm based on local information [23].

V. CONCLUSIONS

In conclusion, we have investigated cascading dynamics on scale-free networks by (i) using a more realistic load-capacity relation and (ii) considering weighted routing strategy. Our main finding is the existence of an optimal weighting scheme for which the network exhibits a maximum degree of robustness against cascading failures and traffic congestion. In particular, for the optimal scheme, the two quantities characterizing the degree of the catastrophic dynamics assume values that are indicative of significant enhancement of the network's resistance to such dynamics. The key to this phenomenon lies in the load distribution. The introduction of optimal weighting is to counterbalance heterogeneity so as to make the load distribution as uniform as possible, reducing significantly the likelihood of the occurrence of the catastrophic dynamics. We expect our finding to be relevant to understanding and enhancing the security of real-world complex networks.

While many networks, especially networks in physical and biological systems, are weighted and it is difficult to change the weight of a link as it may be related with the connectivities of the nodes constituting the link, there are situations where some appropriate weights can be implemented. Examples are communication and computer networks, where the link weights are effectively determined by predesigned traffic protocols. From another perspective, the optimal weighting scheme that we discovered can be regarded as a general principle for figuring out various routes for traffic flows on the network. For example, a weight is equivalent to a distance in the sense that a larger weight corresponds to a longer distance. Similar to the calculation of shortest paths in nonweighted networks, under the weighting scheme, weighted shortest paths can be computed analogously and physical loads are then transmitted along these paths. After all paths have been determined, the weights can be abandoned. An appropriate weighting scheme is thus effectively a statistical guide for the load transmission. Our optimal weighting scheme can be used to find the optimal paths for load traffic to enhance the network robustness.

ACKNOWLEDGMENTS

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[1] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **406**, 378 (2000).
[2] A. E. Motter and Y.-C. Lai, Phys. Rev. E **66**, 065102(R) (2002).
[3] A. E. Motter, Phys. Rev. Lett. **93**, 098701 (2004).
[4] L. Zhao, K. Park, and Y.-C. Lai, Phys. Rev. E **70**, 035101(R) (2004).
[5] L. Zhao, K. Park, Y.-C. Lai, and N. Ye, Phys. Rev. E **72**, 025104(R) (2005).
[6] L. Huang, Y.-C. Lai, and G. Chen, Phys. Rev. E **78**, 036116 (2008).
[7] M. L. Sachtjen, B. A. Carreras, and V. E. Lynch, Phys. Rev. E **61**, 4877 (2000).
[8] V. Jacobson, Comput. Commun. Rev. **18**, 314 (1988); R.

Guimerá, A. Arenas, A. Díaz-Guilera, and F. Giralt, Phys. Rev. E **66**, 026704 (2002).
[9] P. Crucitti, V. Latora, and M. Marchiori, Phys. Rev. E **69**, 045104(R) (2004).
[10] M. Schäfer, J. Scholz, and M. Greiner, Phys. Rev. Lett. **96**, 108701 (2006).
[11] B. Wang and B. J. Kim, Europhys. Lett. **78**, 48001 (2007).
[12] W.-X. Wang and G. Chen, Phys. Rev. E **77**, 026101 (2008).
[13] D.-H. Kim and A. E. Motter, J. Phys. A **41**, 224019 (2008).
[14] P. Holme and B. J. Kim, Phys. Rev. E **65**, 066109 (2002).
[15] R. Pastor-Satorras, A. Vázquez, and A. Vespignani, Phys. Rev. Lett. **87**, 258701 (2001).
[16] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998).
[17] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).
[18] K. Park, Y.-C. Lai, and N. Ye, Phys. Rev. E **70**, 026109 (2004).
[19] O. Allen, *Probability, Statistics and Queueing Theory with Computer Science Applications* (Academic, Boston, 1994).
[20] R. Guimerá, A. Díaz-Guilera, F. Vega-Redondo, A. Cabrales, and A. Arenas, Phys. Rev. Lett. **89**, 248701 (2002).
[21] L. Zhao, Y.-C. Lai, K. Park, and N. Ye, Phys. Rev. E **71**, 026125 (2005).
[22] A. Arenas A. Díaz-Guilera, and R. Guimerá, Phys. Rev. Lett. **86**, 3196 (2001).
[23] W.-X. Wang, B.-H. Wang, C.-Y. Yin, Y.-B. Xie, and T. Zhou, Phys. Rev. E **73**, 026111 (2006); W.-X. Wang, C.-Y. Yin, G. Yan, and B.-H. Wang, *ibid.* **74**, 016101 (2006).